ASSESSING YEAR FOUR PUPILS' LEVELS OF PROFICIENCY IN PERIMETER FORMULAE

Wong Sing Ie
School of Educational Studies
Universiti Sains Malaysia
Penang, Malaysia.
Email: britney@live.com.my

Chew Cheng Meng
School of Educational Studies
Universiti Sains Malaysia
Penang, Malaysia.
Email: cmchew@usm.my

ABSTRACT

This study was designed to develop a multi-strand test for assessing Year Four pupils' levels of proficiency in perimeter formula of a square, rectangle and triangle, respectively. A cross-sectional survey is conducted and cluster sampling is used to select the sample of this study from a population of Year 4 pupils who are studying in the state of Penang, Malaysia and have learned perimeter formulae in school. 600 Year Four pupils (286 males and 314 females) were chosen from six public schools located in Penang. The selected pupils were asked to respond to a multi-strand test that is used for assessing their levels of proficiency in perimeter formulae. The findings show that among the five strands of proficiency in perimeter formulae, the strand of adaptive reasoning shows the highest percentage of pupils who acquired very low proficiency, where 80.7%, 78.0% and 82.5% of the pupils possessed very low proficiency in perimeter formula of a square, rectangle and triangle, respectively. The results also reveal Year 4 pupils are weak in conceptual understanding involving perimeter formulae. 66.3%, 71.8% and 74.2% of the of the pupils acquired very low proficiency in the strand of conceptual understanding in perimeter formula of a square, rectangle and triangle, respectively. However, among the five strands in three perimeter formulae, the strand of procedural fluency obtained the highest percentage of pupils who acquired very high level of proficiency in perimeter formula of a square and triangle, where there are 35.2% and 31.7% of pupils acquired very high proficiency in procedural fluency involving perimeter formula of a square and triangle, respectively. For perimeter formula of a rectangle, the strand of strategic competence (30.5%) obtained the highest percentage of pupils who acquired very high proficiency. Overall, the perimeter formula of a square, rectangle and triangle show the highest percentage of pupils who acquired very low proficiency. The results of this study provide teachers with evidence that the pupils are very weak in conceptual understanding and adaptive reasoning in the perimeter formulae and imply that an emphasis on the strands of conceptual understanding and adaptive reasoning should be given when teaching the perimeter formula of a square, rectangle and triangle.

Keywords: Mathematical proficiency, perimeter formulae, multi-strand test.

Introduction

Measurement has formed a vital part in our daily life and a unique part in mathematics programs (Baroody & Coslick, 1998; Pope, 1994). According to Hart (1984), most of the mathematics teachers had categorized measurement as one of the important topic that needs to be taught in the mathematics curriculum. One of the important formulae related to measurement, namely perimeter formulae have become one of the important and classical mathematical knowledge that must be learnt from school as perimeter formulae are the foundation for understanding other aspects of geometry such as area, volume and mathematical theorems that help students to understand algebra, trigonometry and calculus.

Based on the new Standard Curriculum for Primary Schools (Kurikulum Standard Sekolah Rendah, KSSR), primary pupils in Year Four begin to learn the basic parts of measurement formulae such as finding the perimeter formula of a square, rectangle and triangle. In Year Five, primary pupils learn to calculate the perimeter of composite two-dimensional shapes involving square, rectangle and triangle and solve problems involving perimeter of composite two-dimensional shapes. In Year Six, primary pupils learn to calculate the perimeter of composite two-dimensional shapes involving two or more quadrilaterals (square and rectangle only).

Since the pupils start to learn perimeter formulae in Year Four, it is important to ensure that the pupils are practicing good proficiency in perimeter formulae which will benefit them to continue their study in the area of measurement and able to solve problems related to perimeter formulae effectively. In Malaysia, the proficiency of Year Six pupils in perimeter formulae is assessed through Primary School Achievement Test, also known as Ujian Pencapaian Sekolah Rendah (UPSR). Though the mathematics curriculum in Malaysia has been changed in which it is not solely focused on students' procedural knowledge, Zul and Anas (2013) stated that most of the test items in mathematics are still emphasise on the computational skills and deemphasize on reasoning skills.

However, with the high demand of the society in high proficient mathematician in this era, developing highly mathematical proficient students has becoming one of the concerns among educators and teachers. According to Burkhardt (2001), mathematics performance should be assessed holistically and analytically but not assess only one separate part of the performance. Kilpatrick, Swafford and Findell (2001) had introduced a mathematical proficiency model which consists of five strands that can represent a holistic view of the students' thought regarding the characteristics that constitute proficiency in mathematics. The five strands, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition represent different aspects of a complex whole and all the five strands are all interwoven and interdependent in the development of proficiency in mathematics (Kilpatrick et al, 2001).

In order to develop the new generation which is mathematically proficient in perimeter formulae, it is important to know more specific characteristics about their knowledge in perimeter formulae but not just their computational skills in solving problems. With the adaptation of mathematical proficiency model introduced by Kilpatrick et al. (2001) in assessment, teachers can clearly diagnose the weaknesses of their students in solving problems related to perimeter formulae and develop teaching and learning approach that can enhance the students' proficiency in perimeter formulae. The levels of proficiency in this study comprised of five interwoven strands namely, 1) *conceptual understanding* which refers to the understanding of perimeter formulae of a square, rectangle and triangle; 2) *procedural fluency* which refers to the skill in carrying out procedures to solve problems involving perimeter formula of a square, rectangle and triangle; 3) *strategic competence* which refers to the ability to formulate, represent, and solve mathematical problems involving perimeter formula of a square, rectangle and triangle; 4) *adaptive reasoning* which refers to the capacity of logical thought, reflection, explanation and justification involving perimeter formula of a square, rectangle and triangle and 5) *productive disposition* which refers to the habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy (NRC, 2001). Thus, this study focuses particularly on Year 4 pupils' levels of proficiency in perimeter formulae. The problem statement, purpose and objectives of the study, research design and sampling and instrument for this study are further explicated. Ultimately, findings follow with discussion and conclusion will be discussed in this paper.

Statement of the Problem

It was reported that Year Six pupils did not have a good conceptual understanding and procedural fluency in perimeter formulae (Malaysian Examination Syndicate, 2005). Based on the Malaysian Examination Board, the common mistakes made by Year Six pupils in finding perimeter of a whole diagram consisting of three congruent right-angled triangles with two of the triangles having a common hypotenuse to form a rectangle were: (i) adding the common hypotenuse which is located inside the diagram; or (ii) calculating the perimeter of one right-angled triangle and then multiplying the perimeter by three (Malaysian Examination Syndicate, 2005). Furthermore, the common mistakes made by Year Six pupils in finding the perimeter of composite two-dimensional shapes involving a rectangle and a right-angled triangle were: (i) unable to find the length of sides without given measurement; or (ii) adding the length of the sides which is located inside the diagram (Malaysian Examination Syndicate, 2007). This mistake might reveal that the pupils do not have adequate understanding in perimeter as pupils will encounter difficulties to deduce the length of the side when it was not provided explicitly (Yeo, 2008). In finding the perimeter of a shaded region, the common mistakes that were made by the pupils were: (i) adding the length of all the sides of the diagram; or (ii) adding some of the sides of the shaded region instead of all the sides of the shaded region (Malaysian Examination Syndicate, 2010). The mistakes that were made by the pupils reveal that the pupils do not have high proficiency in perimeter formulae.

Further, according to Malaysia Education Blueprint 2013-2025, the performance shown by Malaysian students in 2007 and 2011 were less satisfactory as compared with their previous performance in 1999 and 2003. In Trends in International Mathematics and Science Study 2011 (TIMSS 2011), for one of the released item related to perimeter formulae (ID_M032116) on finding the perimeter of a square with a given area of 144 cm^2, there are only 43% of Malaysian Form Two students were able to answer it correctly as compared to 79% of their Singaporean counterparts. As a result, Malaysian Form Two students' performance was ranked 25th while their Singaporean counterparts was ranked first. The performance shown by Malaysian students always lay behind the performance shown by Singaporean students might be due to the difference in the context of teaching and learning. According to Zul and Anas (2013), in Malaysia, most of the teaching methods in classroom are geared towards exam preparation which most of the teaching and learning emphasise on the computational skills and deemphasise on thinking and reasoning skills. However, in Singapore, teachers always engage students in more thinking and problem solving tasks when teaching mathematical concepts (Kaur & Yeap, 2009).

In order to achieve Malaysian aspiration to be in top third of countries in terms of performance in TIMSS within 15 years, there is a need to concern on the context of assessment, in which more emphasise should be given on proficiency assessment instead of achievement assessment when learning perimeter formulae (Malaysian Ministry of Education, 2012). Achievement assessment basically required the students to demonstrate their retention of what they have learnt before the exam. However, proficiency assessment enables the teachers to evaluate on the branch of the knowledge and proficiency in perimeter formulae within the pupils. Though there are some researchers studied on secondary students' proficiency in mathematics, yet study on primary school pupils' proficiency, specifically on perimeter formulae is still sparse. Thus, there is a need to develop a multi-strand test to assess Year Four pupils' levels of proficiency in perimeter formulae to provide evidence for teacher regarding their pupils' proficiency in perimeter formulae so that appropriate and effective approach can be designed and implemented in classroom to improve pupils' mathematics performance.

Purpose and Objectives of the Study

The purpose of this study was to develop a multi-strand test for assessing Year 4 pupils' levels of proficiency in perimeter formula of a square, rectangle and triangle, respectively. The specific objectives of this study were:

1. What is the profile of Year 4 pupils' levels of proficiency in perimeter formula of a square?
2. What is the profile of Year 4 pupils' levels of proficiency in perimeter formula of a rectangle?
3. What is the profile of Year 4 pupils' levels of proficiency in perimeter formula of a triangle?

Research design and Sampling

The researcher has applied cross-sectional research design in this research study as this design is effective for systematically collecting data regarding Year 4 pupils' levels of proficiency from a broad spectrum of populations and offers a snapshot at a single moment in time (Martyn, 2010). The sample of this study consisted of 600 Year Four pupils which were selected from six public schools in Penang (100 from each school) through a process of cluster sampling. 314 (52.3%) of them were females and 286 (47.7%) of them were males.

Instrument

A multi-strand test was developed based on the adaptation of the five strands of mathematical proficiency model introduced by Kilpatrick, Swafford and Findell in 2001, and Malaysian Year 4 Mathematics Curriculum and Assessment Standard Document of the Primary School Standard Curriculum to assess Year 4 pupils' levels of proficiency in perimeter formulae. The levels of proficiency in this study comprised of five interwoven strands namely, 1) *conceptual understanding* which refers to the understanding of perimeter formulae of a square, rectangle and triangle; 2) *procedural fluency* which refers to the skill in carrying out procedures to solve problems involving perimeter formula of a square, rectangle and triangle; 3) *strategic competence* which refers to the ability to formulate, represent, and solve mathematical problems involving perimeter formula of a square, rectangle and triangle; 4) *adaptive reasoning* which refers to the capacity of logical thought, reflection, explanation and justification involving perimeter formula of a square, rectangle and triangle and 5) *productive disposition* which refers to the habitual inclination to see mathematics as sensible, useful, worthwhile, coupled with a belief in diligence and one's own efficacy (NRC, 2001).

The multi-strand test is divided into two sections, in which, Section (i) requires the pupils to fill up their name, school, latest mathematics mark, location of school and gender, whereas Section (ii) can be divided into three subtests, where each subtest comprised of five items. The first subtest was developed to assess the levels of proficiency in perimeter formula of a square, the second subtest was developed to assess the levels of proficiency in perimeter formula of a rectangle, and the third subtest was developed to assess the levels of proficiency in perimeter formula of a triangle. In each subtest, the first, second, third, fourth and fifth item were developed to assess the strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, respectively. The fifth item for each subtest comprised of five sub-items to assess the five aspects of productive disposition, namely sensible, useful, worthwhile, diligence and efficacy.

The test was given to a panel of experts which consisted of three experienced primary school mathematics teachers for validation. The instrument was improved based on the comments and suggestions of the panel of experts to ensure that the instrument is well-developed to measure the levels of proficiency in perimeter formulae. The validated test was conducted in the actual test. The responses of each pupil were scored based on the scoring rubric developed by the researcher. The range of the scores for the first, second, third, fourth, and each sub-item for the fifth item are: 0 (Very Low Proficiency); 1 (Low Proficiency); 2 (Average Proficiency); 3 (High Proficiency); and 4 (Very High Proficiency). For the fifth item for each subtest, the scores obtained by the pupil in each sub-item were added up and coded into five different levels, in which pupils score from 0 to 4 were coded as 0 (Very Low Proficiency); 5 to 8 were coded as 1 (Low Proficiency); 9 to 12 were coded as 2 (Average Proficiency); 13 to 16 were coded as 3 (High Proficiency); 17 to 20 were coded as 4 (Very High Proficiency) in the strands of productive disposition in perimeter formulae.

Internal consistency using Cronbach's Alpha was determined to test the reliability of the multi-strand test. According to Nunally (1987), the value of Cronbach's Alpha greater than 0.7 indicates that the instrument is reliable. Thus, based on the results in **Table 1**, the multi-strand test was found to be reliable as the values of Cronbach's Alpha are all greater than 0.7.

Table 1: Cronbach's alpha for the multi-strand test of levels of proficiency in perimeter formulae

Multi-Strand Test of Levels of Proficiency in Perimeter Formulae	Items	Cronbach's Alpha
Perimeter formula of a square	1, 2, 3, 4, 5a, 5b, 5c, 5d, 5e	0.78
Perimeter formula of a rectangle	6, 7, 8, 9, 10a, 10b, 10c, 10d, 10e	0.76
Perimeter formula of a triangle	11, 12, 13, 14, 15a, 15b, 15c, 15d, 15e	0.76

Results

The results of the actual test are discussed to profile Year 4 pupils' levels of proficiency in perimeter formulae according to the research questions of this study. **Table 2** shows the Year 4 pupils' levels of proficiency in perimeter formula of a square. Among the five strands of proficiency, the strand of adaptive reasoning obtained the highest percentage of Year 4 pupils who acquired very low level of proficiency (80.7%) whereas the strand of procedural fluency obtained the highest percentage of pupils who acquired very high level of proficiency (35.2%). Furthermore, the strand of conceptual understanding obtained the lowest percentage of pupils who acquired very high level of proficiency (1.3%) in conceptual understanding involving perimeter formula of a square. Overall, 2.8% of the Year 4 pupils are very high proficienct while 37.2% are very low proficienct in perimeter formula of a square.

Table 3 shows the Year 4 pupils' levels of proficiency in perimeter formula of a rectangle. Among the five strands of proficiency, the strand of adaptive reasoning obtained the highest percentage of Year 4 pupils who acquired very low level of proficiency (78.0%) whereas the strand of strategic competence obtained the highest percentage of pupils who acquired very high level of proficiency (30.5%). In addition, the strand of adaptive reasoning also obtained the lowest percentage of pupils who acquired very low level of proficiency (0.2%). Overall, 1.8% of the Year 4 pupils are very high profcient while 50.7% are very low proficienct in perimeter formula of a rectangle.

Table 4 shows the Year 4 pupils' levels of proficiency in perimeter formula of a triangle. Among the five strands of proficiency, the strand of adaptive reasoning also obtained the highest percentage of pupils who acquired very low level of proficiency (82.5%), whereas the strand of procedural fluency shows the highest percentage of pupils who acquired very high level of proficiency in procedural fluency involving perimeter formula of a triangle (31.7%). Furthermore, the strand of conceptual understanding involving perimeter formula of a triangle obtained the lowest percentage of pupils who acquired very high level of proficiency (1.0%). Overall, there are only 2.0% if the Year 4 pupils who acquired very high proficiency while 47.3% of the pupils who acquired very low proficiency in perimeter formula of a triangle.

Table 2: Levels of proficiency in the perimeter formula of a square

	Conceptual understanding		Procedural fluency		Strategic competence		Adaptive reasoning		Productive Disposition		Overall	
	N	%	N	%	N	%	N	%	N	%	N	%
Very Low proficiency	398	66.3	255	42.5	232	38.7	484	80.7	10	1.7	223	37.2
Low proficiency	79	13.2	5	0.8	4	0.7	19	3.2	37	6.2	104	17.3
Average proficiency	49	8.2	75	12.5	97	16.2	65	10.8	182	30.3	134	22.3
High proficiency	66	11.0	54	9.0	58	9.7	9	1.5	228	38.0	122	20.3
Very high proficiency	8	1.3	211	35.2	209	34.8	23	3.8	143	23.8	17	2.8
Total	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0

Table 3: Levels of proficiency in the perimeter formula of a rectangle

	Conceptual understanding		Procedural fluency		Strategic competence		Adaptive reasoning		Productive Disposition		Overall	
	N	%	N	%	N	%	N	%	N	%	N	%
Very Low proficiency	431	71.8	336	56.0	310	51.7	468	78.0	13	2.2	304	50.7
Low proficiency	60	10.0	7	1.2	5	0.8	47	7.8	39	6.5	83	13.8
Average proficiency	61	10.2	52	8.7	61	10.2	80	13.3	180	30.0	98	16.3
High proficiency	41	6.8	40	6.7	41	6.8	4	0.7	229	38.2	104	17.3
Very high proficiency	7	1.2	165	27.5	183	30.5	1	0.2	139	23.2	11	1.8
Total	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0

Table 4: Levels of proficiency in the perimeter formula of a triangle

	Conceptual understanding		Procedural fluency		Strategic competence		Adaptive reasoning		Productive Disposition		Overall	
	N	%	N	%	N	%	N	%	N	%	N	%
Very Low proficiency	445	74.2	311	51.8	308	51.3	495	82.5	16	2.7	284	47.3
Low proficiency	54	9.0	7	1.2	8	1.3	22	3.7	42	7.0	91	15.2
Average proficiency	48	8.0	56	9.3	64	10.7	64	10.7	194	32.4	123	20.5
High proficiency	47	7.8	36	6.0	40	6.7	2	0.3	215	35.8	90	15.0
Very high proficiency	6	1.0	190	31.7	180	30.0	17	2.8	133	22.2	12	2.0
Total	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0	600	100.0

Discussion

Based on the results in Table 2, 3 and 4, among the five strands of proficiency, the strand of adaptive reasoning shows the highest percentage of pupils who acquired very low level of proficiency in perimeter formula of a square, rectangle and triangle, respectively. Researcher expected that the pupils are weak in this strand as adaptive reasoning involves the application of reasoning skills to justify and explain solutions to a non-routine problem related to perimeter formula of a square, rectangle and triangle. In the multi-strand test, the items which are used to determine the levels of proficiency in adaptive reasoning involving perimeter formula of a square, rectangle and triangle required the pupils to draw as many squares, rectangles and equilateral triangles, respectively, as possible with a perimeter of 12cm and explain their answers. The results show that above 75% of the

Year 4 pupils from the sample obtained very low proficiency in the strand of adaptive reasoning in perimeter formula of a square, rectangle and triangle. These results suggest that the high percentage of pupils who acquired very low level of proficiency in adaptive reasoning might be due to the lacking of understanding and reasoning skills among Year 4 pupils as Schön (1987) stated that the reasoning skills of the children under 12 years old are quite limited.

According to Alexander et al. (1997), students can have a better reasoning skills when students are familiar with the context of the item and have adequate knowledge. The results which show that the Year 4 pupils are weak in adaptive reasoning suggest that the Year 4 pupils might not have adequate understanding and knowledge in perimeter formulae. In addition, the pupils might not familiar with the context of the item or the items involving adaptive reasoning were not understandable for the pupils. Furthermore, in answering the question used to test pupils' levels of proficiency in adaptive reasoning involving perimeter formula of a triangle, in which the question required the pupils to draw as many equilateral triangles as possible with the perimeter of 12cm, some of the pupils provided non-equilateral triangles with perimeter of 12cm. This result shows that Year 4 pupils might not have sufficient knowledge on types of triangles. Thus, more attention should be given while learning of types of triangles to ensure that the pupils can differentiate the types of triangles.

Overall, the results reveal that there is a higher percentage of Year 4 pupils possessed very low proficiency in perimeter formula of a square, rectangle and triangle. A few studies have indicated that some of the trainee teachers were lack of understanding and found that some of them believed that learning mathematics was mainly on procedural skills and focused on the formulae (Cheah, 2001). Based on the mathematical proficiency model introduced by Kilpatrick et al. (2001), mathematical proficiency cannot be achieved by just focusing on one of the strands isolately as all the five strands are intertwoven between each other. The phenomenon in which the teachers believed that procedural skill is the most important skill in learning mathematics might be one of the reasons that caused higher percentage of pupils in acquiring higher level of proficiency in procedural fluency and strategic competence rather than acquiring higher level of proficiency in conceptual understanding and adaptive reasoning. Furthermore, higher percentage of pupils obtained high proficiency in procedural fluency in perimeter formulae might be due to one of the issues that is still happening in the schools, in which pupils always view skill and competence as the centre or the whole of the instruction in the classroom, thus, deemphasize on conceptual understanding and adaptive reasoning (NRC, 2005). Thus, it is suggested that educators and teachers in school should emphasise more on the conceptual understanding and adaptive reasoning skills in schools when teaching and learning mathematics involving perimeter formulae in order to develop pupils that are highly proficient in perimeter formulae.

Conclusion

The objectives of this study is to profile the Year 4 pupils' levels of proficiency in perimeter formulae of a square, rectangle and triangle. It is found that the pupils are weak in the strands of conceptual understanding and adaptive reasoning in perimeter formula of a square, rectangle and triangle. Results show that Year Four pupils have better proficiency in procedural fluency and strategic competence, as these two strands gain the highest percentage of pupils with very high proficiency. This means that pupils should be exposed to the strands of conceptual understanding and adaptive reasoning while learning perimeter formulae in schools. Teachers shall change their teaching and learning materials which can enhance the pupils' understanding in perimeter formulae. Pupils with good conceptual understanding in perimeter formulae enable them to build a strong connection between new knowledge and prior knowledge. With an organized knowledge, pupils are capable of solving non-routine problems, justify and explain their answer. Hence, the strand of adaptive reasoning can be enhanced with a strong conceptual understanding in perimeter formulae as Kenneth (2003) stated that adaptive reasoning has a strong relation with the strand of conceptual understanding.

Besides that, the results of the study indicated the developed multi-strand test is able to assess pupils' levels of proficiency in perimeter formula of a square, rectangle and triangle. It is hoped that more multi-strand test involving other mathematical topics involving area and volume formulae can be developed to provide teachers as an instrument for assessing each strand of pupils' mathematical proficiency in measurement formulae. It is recommended that teachers should give assessment using multi-strand test which enable teachers to diagnose each strands of their pupils' proficiency accurately. The suggested practice is believed to be effective in helping teachers to improve their teaching instructions and thus, enhance pupils' mathematical proficiency in perimeter formulae.

However, the limitation of this study was that the sample involved were Year Four pupils from six primary schools in Penang state only. Therefore, the results of this study could not be generalized to other Year Four pupils' levels of proficiency in perimeter formulae and could not represent all primary schools in Malaysia.

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